Polarization splitter of surface polaritons

Qiang Bai,¹ Jing Chen,² Cong Liu,¹ Ji Xu,¹ Chen Cheng,¹ Nian-Hai Shen,¹ and Hui-Tian Wang^{1,2[,*](#page-5-0)}

Department of Physics and Nanjing National Laboratory of Microstructures, Nanjing University, Nanjing 210093,

People's Republic of China

²*School of Physics, Nankai University, Tianjin 300071, People's Republic of China*

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We investigate surface polaritons (SPs) at the interface associated with anisotropic metamaterials. The existence conditions and handednesses of *s*- and *p*-polarized SPs are discussed in detail, which reveal that anisotropy is necessary for coexistence of *s*- and *p*-polarized SPs. We explore the reflection and transmission properties of SPs across the boundary separating two interfaces and then find the scaling relations that ensure the elimination of the parasite out-of-plane scattering of SPs. The reflection and transmission coefficients, the Brewster's angle, and the critical angle of total reflection are developed. We propose a compact kind of polarization SP splitter, and its functional characters can be tuned by artificially designed constitutive parameters.

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I. INTRODUCTION

Surface polaritons (SPs), as a kind of surface electromagnetic (EM) waves, are highly confined at the interface separating two media and decay exponentially from the interface.^{1[,2](#page-5-2)} It has attracted extensive attention because of various important applications[.3](#page-5-3) Dispersion property of SPs can be modified by changing the dielectric function of the media forming the interface, λ e.g., in semiconductors by changing the doping density⁴ or by using external optical excitation,⁵ (in particular, the latter can provide an *in situ* tuning possibility). In addition, its group velocity is greatly reduced around resonance and can approach the drift velocity of electrons. The matched group velocity provides a necessary condition for the exchange of information between photons and charge-density wave.⁶

As a promising subject that leads to various applications, much attention has been paid to the excitation, propagation, and scattering of SPs. It is well known that incident light can effectively launch SPs in metallic slabs with subwavelength structures and can tunnel through the slab or travel a long distance along the interface, and vice versa. $7-10$ $7-10$ In addition, plasmonic waveguides, $\frac{11,12}{2}$ $\frac{11,12}{2}$ $\frac{11,12}{2}$ plasmonic Bragg reflectors, $\frac{13}{2}$ plasmonic lenses, 14 and plasmonic light sources¹⁵ are also investigated, as reviewed in Refs. [1](#page-5-1) and [7.](#page-5-7)

Until now p -polarized SPs $(p$ SPs) have been intensively investigated since only negative permittivity is accessible at optical frequency in nature. Recently, metamaterials with negative elements of permittivity $\bar{\bar{\epsilon}}$ and/or permeability $\bar{\bar{\mu}}$ tensors have attracted increasing interest because of their pe-culiar EM properties.^{16,[17](#page-5-15)} The interface associated with metamaterials can support both *p*- and *s*-polarized SPs *(s*) SPs). $18-21$ $18-21$

In this paper, we propose an idea, which uses anisotropic metamaterials to achieve a polarization splitter of SPs. The anisotropic metamaterials can be obtained from layered isotropic materials[.16](#page-5-14)[,22–](#page-5-18)[24](#page-5-19) We first study the existence conditions and handednesses of *s* and *p* SPs at an interface associated with anisotropic metamaterials. Our results show that anisotropy is a necessary requirement for the coexistence of *s* and *p* SPs at the same frequency. To realize the polarization splitter of SPs in the true surface two-dimensional (2D) optics, we develop the relations among the elements of $\bar{\mu}$ and $\bar{\varepsilon}$ tensors. The relations ensure the elimination of the parasite out-of-plane scattering of both polarizations, which happens when SP travels from an interface to the other one. The relations we developed are more universal than those discussed in Ref. [25,](#page-5-20) in which only the *p*-polarized SP is considered. With the aid of the expressions of reflection and transmission coefficients, the Brewster's angle θ_R , and the critical angle of total reflection θ_C , we finally discuss how to choose the proper parameters, which enable the SP splitter to reflect completely one polarization of SP while transmit the other polarization of SP freely.

II. EXISTENCE CONDITIONS

We first discuss the existence conditions of SPs. Let us consider a single interface as illustrated in Fig. [1.](#page-1-0) The interface in the $yz(x=0)$ plane separates two media 1 $(x>0)$ and $2(x<0)$. For the sake of simplicity, here we assume that the medium *j* has its permittivity and permeability tensors, $\bar{\bar{\epsilon}}_j$ and $\bar{\bar{\mu}}_i$, as follows:

$$
\bar{\varepsilon}_j = \begin{bmatrix} \varepsilon_j^x & 0 & 0 \\ 0 & \varepsilon_j^{yz} & 0 \\ 0 & 0 & \varepsilon_j^{yz} \end{bmatrix},
$$
(1a)

$$
\bar{\mu}_{j} = \begin{bmatrix} \mu_{j}^{x} & 0 & 0 \\ 0 & \mu_{j}^{yz} & 0 \\ 0 & 0 & \mu_{j}^{yz} \end{bmatrix},
$$
 (1b)

where $j=1$ and 2 stand for media 1 and 2, respectively. Evidently, each medium exhibits identical EM property in the *y* and *z* directions (that is to say, isotropic in the *yz* plane), and the x direction is the uniquely distinguishable principal axis[.26](#page-5-21) This assumption ensures that the property of SP in the $x=0$ interface is independent of the propagation direction. Throughout this paper, we adopt $(e_j^x \text{ and } e_j^{yz}; \mu_j^x \text{ and } \mu_j^{yz})$ to describe the EM property of medium *j*.

FIG. 1. (Color online) Schematic illustration of the interface between two anisotropic metamaterials and the coordinate system.

Without loss of generality, we first deal with the *s* SPs propagating along the *z* direction, with the electric field parallels to the y direction (For p SPs, results can be yielded similarly with the EM duality). The electric components can be written as

$$
\mathbf{E}_{1(2)} = \hat{\mathbf{y}}A_{1(2)}^0 \exp(i\beta z \pm \alpha_{1(2)}x), \tag{2}
$$

where \hat{y} denotes the unit vector in the *y* direction, β is the propagation constant, and $A_{1(2)}^0$ is the complex amplitude of field inside medium 1 (2). $\alpha_{1(2)}$ is positive, and the +(-) sign in front of $\alpha_{1(2)}$ ensures that the EM field decays exponentially away from the interface at $x=0$. By substituting Eq. ([2](#page-1-1)) into Maxwell's equations and with the aid of EM boundary conditions, the dispersion relation of *s* SP is given by

$$
0 = \alpha_1 \mu_2^{yz} + \alpha_2 \mu_1^{yz},\tag{3}
$$

$$
\beta^2 = \frac{\omega^2}{c^2} \frac{\mu_1^x \mu_2^x (\varepsilon_1^{yz} \mu_2^{yz} - \varepsilon_2^{yz} \mu_1^{yz})}{\mu_2^x \mu_2^{yz} - \mu_1^x \mu_1^{yz}},
$$
(4)

$$
\alpha_{1(2)}^2 = \frac{\omega^2}{c^2} (\mu_{1(2)}^{yz})^2 \frac{\varepsilon_1^{yz} \mu_1^x - \varepsilon_2^{yz} \mu_2^x}{\mu_2^{yz} \mu_2^x - \mu_1^{yz} \mu_1^x},
$$
(5)

where c is the speed of light in vacuum. The time-averaged Poynting vector **S** is

$$
\mathbf{S} = \boldsymbol{\beta} \frac{|A_0|^2}{4\omega\mu_0} \bigg(\frac{1}{\mu_2^x \alpha_2} + \frac{1}{\mu_1^x \alpha_1} \bigg), \tag{6}
$$

where $\beta = \beta \hat{z}$ (\hat{z} denotes the unit vector in the *z* direction). Equation ([3](#page-1-2)) reveals evidently that μ_1^{yz} and μ_2^{yz} must have opposite signs $(\mu_1^{yz}\mu_2^{yz} < 0)$ for ensuring the presence of *s* SP. For the case of *p* SP, $\varepsilon_1^{yz}\varepsilon_2^{yz}$ < 0 is required.

Here we only consider two universal cases for the signs of the tensor elements of $\bar{\bar{\epsilon}}_j$ and $\bar{\bar{\mu}}_j$. For the first case (case I), ε_1^x , ε_2^{yz} , μ_1^x , and μ_1^{yz} are all positive; while ε_2^x , ε_2^{yz} , μ_2^x , and μ_2^{yz} are all negative. In the second case (case II), $\varepsilon_1^{\bar{x}}, \varepsilon_1^{\bar{y}\bar{z}}, \mu_2^{\bar{x}},$ and μ_2^{yz} are positive; while μ_1^x , μ_1^{yz} , ε_2^x , and ε_2^{yz} are negative.

In case I, the existence of *s* SP requires that β , α_1 , and α_2 are real and positive. With Eqs. (3) (3) (3) – (5) (5) (5) , we can readily find two possibilities as follows:

$$
LH: \frac{\varepsilon_2^{yz}}{\varepsilon_1^{yz}} < \frac{\mu_1^x}{\mu_2^x} < \frac{\mu_2^{yz}}{\mu_1^{yz}}, \quad \text{with } S \cdot \beta < 0,
$$
 (7a)

TABLE I. The coexistence conditions and SP handedness for *s* SP and *p* SP in case I.

	Situation Polarization	Existence condition	SP handedness
A	s SP	$\frac{\varepsilon_2^{yz}}{\varepsilon_1^{yz}} \leq \mu_1^x / \mu_2^x \leq \mu_2^{yz} / \mu_1^{yz}$	LH
	p SP	$\mathcal{E}_2^{yz}/\mathcal{E}_1^{yz} \leq \mathcal{E}_1^x/\mathcal{E}_2^x \leq \mu_2^{yz}/\mu_1^{yz}$	RH
B	S SP	$\epsilon_2^{yz}/\epsilon_1^{yz} > \mu_1^x/\mu_2^x > \mu_2^{yz}/\mu_1^{yz}$	RH
	p SP	$\epsilon_2^{yz}/\epsilon_1^{yz} > \epsilon_1^{x}/\epsilon_2^{x} > \mu_2^{yz}/\mu_1^{yz}$	LН

$$
RH: \frac{\varepsilon_2^{yz}}{\varepsilon_1^{yz}} > \frac{\mu_1^x}{\mu_2^x} > \frac{\mu_2^{yz}}{\mu_1^{yz}}, \text{ with } S \cdot \beta > 0. \tag{7b}
$$

In Eq. $(7a)$ $(7a)$ $(7a)$, since β and **S** are in the counter direction, the *s* SP is left handed (LH), while with Eq. ([7b](#page-1-5)) it is right handed $(RH).^{27}$ $(RH).^{27}$ $(RH).^{27}$

With the EM duality, the existence conditions of *p* SP can be readily obtained as

$$
LH: \frac{\mu_2^{yz}}{\mu_1^{yz}} < \frac{\varepsilon_1^x}{\varepsilon_2^x} < \frac{\varepsilon_2^{yz}}{\varepsilon_1^{yz}}, \text{ with } S \cdot \beta < 0,
$$
 (8a)

$$
RH: \frac{\mu_2^{yz}}{\mu_1^{xz}} > \frac{\varepsilon_1^x}{\varepsilon_2^x} > \frac{\varepsilon_2^{yz}}{\varepsilon_1^{yz}}, \text{ with } S \cdot \beta > 0. \quad (8b)
$$

We can see that *s* SP and *p* SP can be supported at the same interface and at the same frequency, provided that Eqs. $(7a)$ $(7a)$ $(7a)$ and $(8b)$ $(8b)$ $(8b)$ [or Eqs. $(7b)$ $(7b)$ $(7b)$ and $(8a)$ $(8a)$ $(8a)$] are satisfied simultaneously. As listed in Table [I,](#page-1-8) for case I, the coexistence of both polarizations requires that the handedness of the *s* SP differs from that of the *p* SP.

Similarly, we readily validate that *s* SP and *p* SP can also coexist in case II, but with the same handedness. The existence conditions and the handedness are listed in Table [II.](#page-1-9)

From the above analysis, it is clear that in the two cases the coexistence of both polarizations at a given frequency requires that at least one of the two media is anisotropic. When one medium is isotropic, for example, $\varepsilon_1^x = \varepsilon_1^{yz}$ and $\mu_1^x = \mu_1^{yz}$, then $\varepsilon_2^x \neq \varepsilon_2^{yz}$ and/or $\mu_2^x \neq \mu_2^{yz}$ are required, implying that the other medium must be anisotropic. It is the reason why the coexistence of *s* SP and *p* SP was not found in previous publications such as Refs. [19](#page-5-23) and [20](#page-5-24) in which the considered media are all isotropic.

In Fig. [2,](#page-2-0) we show an example of coexisted *s* SP and *p* SP in which medium 1 is vacuum while the anisotropic metamaterial 2 has the tensor elements with the dispersions of ε_{2}^{yz} $= 1 - 160/f^2$, $\varepsilon_2^x = 1 - 18/f^2$, $\mu_2^{yz} = 1 - 48/f^2$, and $\mu_2^x = 1 - 20/f^2$

TABLE II. The conditions and SP handedness for the coexistence of *s* SP and *p* SP in case II.

	Situation Polarization	Existence condition	SP handedness
A	s SP	$\mu_1^x/\mu_2^x \leq \mu_2^{yz}/\mu_1^{yz} \leq \varepsilon_2^{yz}/\varepsilon_1^{yz}$	RН
	p SP	$\mu_2^{yz}/\mu_1^{yz} \leq \varepsilon_2^{yz}/\varepsilon_1^{yz} \leq \varepsilon_1^{x}/\varepsilon_2^{x}$	RH
В	S SP	$\epsilon_2^{yz}/\epsilon_1^{yz} < \mu_2^{yz}/\mu_1^{yz} < \mu_1^{x}/\mu_2^{x}$	LH
	p SP	$\epsilon_1^x/\epsilon_2^x \leq \epsilon_2^{yz}/\epsilon_1^{yz} \leq \mu_2^{yz}/\mu_1^{yz}$	LH

FIG. 2. (Color online) Dispersions curves of *s* SPs and *p* SPs. Medium 1 is vacuum, while anisotropic metamaterial 2 has dispersions of $\varepsilon_2^{yz}=1-160/f^2$, $\varepsilon_2^{x}=1-18/f^2$, $\mu_2^{yz}=1-48/f^2$, and μ_2^{x} $= 1 - 20/f^2$ in which the frequency *f* is the unit of GHz.

(where the frequency f is in unit of GHz). The resonant frequency of s (p) polarization can be found from Eq. ([4](#page-1-10)) by $\mu_2^x \mu_2^{yz} - \mu_1^x \mu_1^{yz} = 0$ ($\varepsilon_2^x \varepsilon_2^{yz} - \varepsilon_1^x \varepsilon_1^{yz} = 0$) and is 3.757 (4.022) GHz. Therefore, within the frequency range of 3.757 GHz < f $<$ 4.022 GHz, both *s* SP and *p* SP exist (but they might have different wave vectors). The slope of the s SP $(p$ SP) dispersion curve is negative (positive), implying that the *s* SP is LH (the p SP is RH), which corresponds to situation A of case I listed in Table [I.](#page-1-8)

III. REFLECTION AND TRANSMISSION

It is then very interesting to explore the reflection and transmission behaviors of SP. As shown in Fig. [3,](#page-2-1) two interface structures are separated by the boundary $z=0$. The interface structure composed of media 1*L* and 2*L* occupies the $z<0$ space, while the other one consisting of media 1*R* and 2*R* lies in the $z > 0$ space. The two interfaces are all in the $x=0$ plane and normal to the *x* direction. The EM parameters of the four media 1*L*, 2*L*, 1*R*, and 2*R* are $(e_{1L}^x$ and e_{1L}^{yz} ; μ_{1L}^x and μ_{1L}^{yz} , $(\varepsilon_{2L}^x$ and ε_{2L}^{yz} ; μ_{2L}^x and μ_{2L}^{yz} , $(\varepsilon_{1R}^x$ and ε_{1R}^{yz} ; μ_{1R}^x and μ_{1R}^{yz}), and (ε_{2R}^{x}) and ε_{2R}^{yz} ; μ_{2R}^{x} and μ_{2R}^{yz}), respectively, as labeled in Fig. [3.](#page-2-1)

To achieve the perfect (lossless) reflection and transmission of SPs at the boundary $z=0$ (such as those of the tradi-

FIG. 3. (Color online) Schematic illustration of the SP splitter and the coordinate system.

tional volume EM wave at the interface separating two dielectric materials), two requirements should be satisfied: (i) the boundary $z=0$ does not support any SP mode, and (ii) the conversion from the SP mode to the out-of-plane volume modes must be prohibited. To satisfy requirement (i), the corresponding tensor elements of the two adjacent media across the boundary $z=0$ should have the same signs, i.e., $\varepsilon_{1L}^x \varepsilon_{1R}^x > 0$, $\varepsilon_{1L}^y \varepsilon_{1R}^{yz} > 0$, $\varepsilon_{2L}^x \varepsilon_{2R}^x > 0$, $\varepsilon_{2L}^{yz} \varepsilon_{2R}^{yz} > 0$, $\mu_{1L}^x \mu_{1R}^x > 0$, $\mu_{1L}^{yz} \mu_{1R}^{yz} > 0$, $\mu_{2L}^{x} \mu_{2R}^{yz} > 0$, and $\mu_{2L}^{yz} \mu_{2R}^{yz} > 0$. As for requirement (ii), it is well known that in general, when a SP travels from an interface to another one, part of the EM energy gets lost by scattering into the out-of-plane volume modes. $28,29$ $28,29$ Especially, when a SP is incident into an abrupt free space boundary, 10–30% of SP energy is converted into the volume modes.²⁹ Requirement (ii) can be satisfied only when the spatial profiles of SPs at the two different interfaces match with each other²⁵ as

$$
\alpha_{1L} = \alpha_{1R} \quad \text{and} \quad \alpha_{2L} = \alpha_{2R}, \tag{9}
$$

where α_{1L} , α_{2L} , α_{1R} , and α_{2R} have the similar expressions to Eq. (5) (5) (5) .

Equation (9) (9) (9) can also be proved by directly studying the reflection and transmission of a SP mode from the boundary $z=0$, with the EM boundary conditions for all the field components. Like the traditional volume EM wave, here we define the refractive index of *s* SP as $n^s = \beta c / \omega$. Let us consider the *s* SP supported by the interface $x=0$ in the regime $z<0$. This *s* SP is incident onto the boundary $z=0$ with its propagation direction having an angle of incidence θ_i with respect to the *z* direction. The reflected *s* SP in the regime $z < 0$ and the transmitted *s* SP in the regime $z > 0$ have an angle of reflection θ_r and transmission θ_t , respectively. By considering the EM boundary conditions for all the field components, the scaling relations among the tensor elements can be found, as follows:

$$
\frac{\mu_{1R}^{yz}}{\mu_{1L}^{yz}} = \frac{\mu_{2R}^{yz}}{\mu_{2L}^{yz}} = \left(\frac{\varepsilon_{1R}^{yz}}{\varepsilon_{1L}^{yz}}\right)^{-1} = \left(\frac{\varepsilon_{2R}^{yz}}{\varepsilon_{2L}^{yz}}\right)^{-1} = s_1,
$$
 (10a)

$$
\frac{\mu_{1R}^x}{\mu_{1L}^x} = \frac{\mu_{2R}^x}{\mu_{2L}^x} = s_2.
$$
 (10b)

The angles of incidence θ_i , reflection θ_r , and transmission θ_i satisfy the relations of

$$
n_L^s \sin(\theta_i) = n_L^s \sin(\theta_r) = n_R^s \sin(\theta_t), \qquad (11a)
$$

where

$$
\left(\frac{n_R^s}{n_L^s}\right)^2 = \frac{s_2}{s_1}.\tag{12a}
$$

The reflection and transmission coefficients for *s* SP, *r^s* and *t s* , are

$$
r^{s} = \frac{\cos(\theta_{i}) - \sqrt{s_{1}s_{2}}\cos(\theta_{i})}{\cos(\theta_{i}) + \sqrt{s_{1}s_{2}}\cos(\theta_{i})}, \quad t^{s} = \frac{2\sqrt{s_{1}s_{2}}\cos(\theta_{i})}{\cos(\theta_{i}) + \sqrt{s_{1}s_{2}}\cos(\theta_{i})}.
$$
\n(13a)

Compared with the standard formula for the traditional volume EM wave in isotropic dielectric media,³⁰ $\varepsilon_R/\varepsilon_L$ is replaced by $(\epsilon_{2R}^{yz}/\epsilon_{2L}^{yz})$ (μ_{2L}^x/μ_{2R}^x) .

For *p* SP, we easily obtain the following equations besides Eq. $(10a)$ $(10a)$ $(10a)$:

$$
\frac{\varepsilon_{1L}^x}{\varepsilon_{1R}^x} = \frac{\varepsilon_{2L}^x}{\varepsilon_{2R}^x} = s_3,\tag{10c}
$$

$$
n_L^p \sin(\theta_i) = n_L^p \sin(\theta_r) = n_R^p \sin(\theta_t), \qquad (11b)
$$

$$
\left(\frac{n_R^p}{n_L^p}\right)^2 = \frac{s_1}{s_3},\tag{12b}
$$

$$
r^{p} = \frac{\sqrt{s_1 s_3} \cos(\theta_i) - \cos(\theta_t)}{\sqrt{s_1 s_3} \cos(\theta_i) + \cos(\theta_t)}, \quad t^{p} = \frac{2 \cos(\theta_i)}{\sqrt{s_1 s_3} \cos(\theta_i) + \cos(\theta_t)},
$$
\n(13b)

where $\varepsilon_L / \varepsilon_R$ is replaced by $(\varepsilon_{2L}^x / \varepsilon_{2R}^x)$ $(\mu_{1R}^{yz} / \mu_{1L}^{yz})$.^{[30](#page-5-27)}

Since the signs of the corresponding tensor elements across the boundary $z=0$ are not changed [from requirement (i)], s_1 , s_2 , and s_3 are all positive valued. If we set the permeability $\bar{\mu}$ to be an unity tensor for all the four media, i.e., $\mu^{yz} = \mu^{x} = 1$, the formulas developed above will degenerate into the relations proposed in Ref. [25.](#page-5-20)

One can see from the above results that the reflection and transmission properties of SPs are determined solely by the three scaling parameters, s_1 , s_2 , and s_3 . The properties of *s* SPs depend on s_1 and s_2 , while s_1 and s_3 determine the properties of *p* SPs. s_1 is related to ε^{yz} and μ^{yz} , while s_2 (s_3) is related to μ^x (ε^x) only. Thus we can control the properties of *s* SPs and *p* SPs either simultaneously by tuning s_1 or independently by choosing proper values of s_2 and s_3 , respectively.

To intuitively show those properties, as examples, Fig. [4](#page-3-0) plots the reflectivity $R = |r|^2$ as a function of the angle of incidence θ_i at different s_1 (=4, 8, and 16), when s_2 =3 and $s_3 = 2$. We can find that there is a special angle of incidence, corresponding to the case that the SP is perfectly transmitted. This angle of incidence can be referred to as the Brewster's angle θ_R such as the Brewster's effect of the traditional volume EM wave at the dielectric interface. From Eq. (13) (13) (13) , the Brewster's angles for *s* SPs and *p* SPs can be expressed, respectively, as follows:

$$
\sin^2(\theta_B^s) = \frac{1 - s_1 s_2}{1 - s_1^2}, \quad \sin^2(\theta_B^p) = \frac{s_1 (1 - s_1 s_3)}{s_3 (1 - s_1^2)}.
$$
 (14)

For *s* SPs shown in Fig. [4](#page-3-0)(a), θ_B^s are 0.327, 0.207, and 0.141 π , respectively, for $s_1 = 4$, 8, and 16. For *p* SPs shown in Fig. [4](#page-3-0)(b), θ_B^p are 0.417, 0.430, and 0.447 π , respectively, when $s_1 = 4$, 8, and 16.

FIG. 4. (Color online) Influence of the scaling factor s_1 on the reflectivity *R*, for (a) *s* SPs and (b) *p* SPs, with $s_2 = 3$ and $s_3 = 2$.

As the angle of incidence θ_i increases, *R* decreases slowly toward zero within the range of $\theta_i < \theta_B$ while increases rapidly within the range of $\theta_i > \theta_B$, as shown in Fig. [4.](#page-3-0) When the angle of incidence θ_i exceeds a certain special angle θ_c , R is 100% and the so-called total reflection takes place. θ_c is referred to as the critical angle of total reflection and can be expressed as

$$
\sin^2 \theta_C^s = s_2/s_1, \quad \sin^2 \theta_C^p = s_1/s_3. \tag{15}
$$

For *s* SPs shown in Fig. [4](#page-3-0)(a), θ_C^s are 0.333, 0.210, and 0.143 π , respectively, for $s_1 = 4$, 8, and 16. For *p* SPs, however, no total reflection can be found in Fig. $4(b)$ $4(b)$ due to the fact that $s_1 > s_3$ (if $s_1 < s_3$, θ_C^p certainly exists).

As another example, Fig. [5](#page-4-0) shows the situations at different values of s_2 (=2, 3, and 4), when $s_1 = 5$ and $s_3 = 2$. We can see that the properties of reflection for *s* SPs depend strongly on s_2 , including the values of θ_B^s and θ_C^s as given by Eqs. ([14](#page-3-2)) and (15) (15) (15) , respectively. In contrast, the change in s_2 has no influence on the properties of reflection for *p* SPs. Similar simulations can validate that s_3 only modulates the properties of reflection for *p* SPs.

The existence of θ_B and θ_C as well as their relative positions depend on the values of s_1 , s_2 , and s_3 . From the above examples, we can see that the scaling factor s_1 has the influence on both *s* SPs and *p* SPs, while s_2 (s_3) has the influence

FIG. 5. (Color online) Influence of the scaling factor s_2 on the reflectivity *R* of SPs, with $s_1 = 5$ and $s_3 = 2$.

on s SPs $(p$ SPs) only. Thus we can flexibly control the properties of reflection and transmission for different polarizations as potential applications such as in SP splitters.

IV. SP SPLITTERS

An ideal SP splitter should totally reflect SPs with one special polarization, while completely transmitting SPs with the other polarization, at the same angle of incidence θ_i . Without loss of universality, let us consider the situation that *s* SP is completely transmitted while *p* SP is totally reflected.

Inasmuch as the boundary $z=0$ is opaque for p SPs, the angle of incidence θ_i should be larger than the critical angle of total reflection of *p* SP, i.e., $\theta_i > \theta_C^p$. The existence of θ_C^p then asks $s_1 < s_3$. In other words, $n_R^p < n_L^p$, implying that *p* SP is incident from an optically denser interface to an optically thinner one.

For the total transmission of *s* SP, there are two possibilities. The first one, maybe the most interesting one, is that *s* SP is always totally transmitted at any angle of incidence (so-called omnidirectional total transmission). With Eq. ([13a](#page-3-1)), this situation requires $\theta_t = \theta_i$ and $s_1 s_2 = 1$ at any angle of incidence. Equation ([11a](#page-2-4)) and ([12a](#page-2-5)) give rise to $s_1 = s_2$ $= 1$; thus the realization of this situation requires

$$
s_1 = s_2 = 1 < s_3. \tag{16}
$$

Figure [6](#page-4-1) is an example of such a special situation, with s_1 $= 1$, $s_2 = 1$, and $s_3 = 12$. Since θ_C^p can be very small if $s_1 \ll s_3$, the angle operating range of the SP splitter can be very wide, and the splitter has a good tolerance over any error and per-turbation in the angle of incidence. In Fig. [6,](#page-4-1) due to θ_C^p $\sim 0.093\pi$, the operating range of the SP splitter is from 0.093 to 0.5π , covering 81.4% of all possible angles of incidence. It should be noted that such a special situation is different from that studied by Elser and Podolskiy²⁵ (only p SPs can exist) as the structure we are interested here always support both polarized SPs, with the coexistence requirements listed in Table [I](#page-1-8) or Table [II.](#page-1-9)

In the second situation, the angle of incidence θ_i is just the Brewster's angle θ_B^s as expressed in Eq. ([14](#page-3-2)). Then the SP splitter works if $\theta_B^s \geq \theta_C^p$ so that *s* SP is totally transmitted

FIG. 6. (Color online) Reflective properties of the SP splitter with $s_1 = 1$, $s_2 = 1$, and $s_3 = 12$. The *s* SP is freely transmitted in all angles of incidences.

while *p* SP is totally reflected. Thus the requirements on the scaling factors are either

$$
s_1 < s_3 \cap s_1 < 1 \cap s_1 < s_2 \le \frac{s_3 - s_1(1 - s_1^2)}{s_3 s_1},\tag{17}
$$

or

$$
1 < s_1 < s_3 \cap \frac{s_3 - s_1(1 - s_1^2)}{s_3 s_1} \le s_2 < s_1. \tag{18}
$$

In the situation when Eq. $(17)/Eq.$ $(17)/Eq.$ $(17)/Eq.$ (18) (18) (18) is satisfied, *s* SP is incident from an optically thinner/denser interface to an optically denser/thinner one due to $n_R^s/n_L^s = \sqrt{s_2/s_1}$. We would not discuss this situation here in detail because the SP splitter can work at only one angle of incidence.

From the above analysis, we can see that the critical angle of total reflection and the Brewster's angle can be arbitrarily changed by choosing the proper values of s_1 , s_2 , and s_3 . The splitting angle between the totally transmitted and reflected SPs with different polarizations could then be tuned in a large range, from an acute angle to an obtuse angle. The independent parameter sets for different polarizations enable us to control *s* SP and *p* SPs separately, which allows us to flexibly design various SP elements, including not only the SP splitters discussed in this paper but also other possible ones such as switches and reflectors.

The anisotropic metamaterials discussed in this paper can be easily obtained by the effective medium theory of layered isotropic materials, $16,22-24$ $16,22-24$ $16,22-24$ when the thickness of each layer is much less than the wavelength. The absorption effect of anisotropic metamaterial is neglected in the present work. Stockman 31 argued that it is impossible to eliminate or significantly reduce the loss in double negative medium (with ε < 0 and μ < 0 as considered in case I) by any means, including the compensation by active (gain) media. This issue is still under debate, $32-34$ $32-34$ and at least the single negative medium (with $\epsilon \mu$ <0) discussed in case II could be effectively made lossless by introducing gain mechanism 35 or by di-rectly using dielectric-based structures.^{36[,37](#page-5-33)}

V. CONCLUSION

In conclusion, we study the SPs propagating at the interface formed by anisotropic metamaterials. We find the existence conditions of *s* SP and *p* SP as well as their handedness and show that the anisotropy is necessary for their coexistence. Then we study the reflection and transmission properties of SPs across the boundary separating two interfaces. The reflection and transmission properties of different polarized SPs can be controlled separately. A polarization SP

*htwang@nju.edu.cn/htwang@nankai.edu.cn

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splitter is proposed. By artificially designing the constitutive parameters, the operating properties of this kind of SP splitters can be tuned.

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